Research & Development

A REMARKABLE PRIMES' CONSTANT ASSOCIATED TO NUMBER THEORY

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INTRODUCTION

In ordinary and superior mathematics many advances have been conducted in the domain of measurement and order, since the discovery and the "demonstration of existence", by the greek mathematician Euclid (328-265 BC) in Book IX of his *Elements*, with the elegant theorem which states that "prime numbers are an infinite successions or series".

New concepts, methods and techniques let solve ancient and complex problems related to the prime numbers, their formation and the criteria to decide if any number is or is not a prime. But, the reciprocal is apparently impossible, i.e: nobody has yet shown a simple arithmetic formula to generate prime numbers, although the modern theory of numbers has appealed to radical means (see DECODING THE SECRETS OF THE LAW OF PRIMES IN NUMBER THEORY, La Paz-Bolivia, 10/XI/01, by the same author of this research in pure mathematics).

THE STRUCTURE OF ARITHMETIC

The advances made by the greek mathematicians in relation to the decimal system of numeration let them define the concept of a prime: "is the number which is divisible by itself and 1". Eratosthenes (284-192 bC) built his famous *sieve* to, intuitively and later formally, systematize a procedure which lets find all prime numbers, with the assumption of devoting hard work and enough time, under a simple reasoning and yielding an easy expression.

With the formula n+1 all positive integers are listed in their natural order: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25,…: then the multiples of the successive numbers are eliminated by the proper divisors, except the two trivial divisors "by itself and 1", hence obtaining the series of prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97,…with this siege.

METHODOLOGY OF THE PRESENT RESEARCH

A first conjecture to show states that the prime succession or series becomes scarce as it advances in its development with larger gaps between successive primes, without letting foresee any regularity in the decimal numerical field of study.

It is a laborious process. It has been estimated that over 300 hours of hard work are needed to "sieve" all prime numbers comprised between 1 and 1.000.000, although, observing that they get scarce with larger and larger gaps which, naturally, leads to the question or hypothesis: Is there a maximum prime? or, Do twin primes exist no more somewhere along the successions or series? Indeed, a real challenge for professionals and amateurs.

Building up the Model

With a similar reasoning we ask ourselves, how many prime numbers exist by ranks of number of digits, under the assumption of order in the "positional notation" or "the relative value of the figures", developed by the hindu (beginning of VI century BC) and recognized as one of the major advances of pure and applied mathematics, as the science of measurement and order?.

The formal procedure developed is:

Definition: If $P_n = \{p \text{ primes} / 10^n < p < 10^{n+1}\}$

$P_0 = \{2, 3, 5, 7\}$

$P_1 = \{11, 13, \ldots, 97\}$

$\# P_n = C_n$

where: $C_o = 4$

$C_1 = 21$

$C_2 = 143$

$C_3 = 1061$

:

:

then: $$CRAA - 2 = \frac{1}{C_0} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

in short the formula becomes: $$\sum_{n=0}^{\infty} \frac{1}{C_n}$$

Convergence: the succession or series is convergent because

$$S_m = \sum_{n=0}^{m} \frac{1}{C_n} \quad \text{goes to } CRRA - 2 \in \Re$$

$$if \quad m \to \infty$$

In other words exist

$$\lim_{m \to \infty} S_m$$

Monotonicity: furthermore $\frac{1}{C_n} < \frac{1}{C_m}$ if $m > n$

Which yields a rapid asymptotic curve towards zero (see figure 1)

Scarcity: $$K_n = \frac{\left(10^{n+1} - 10^n\right)}{C_n}$$

Hypothesis $\frac{1}{K_n} << \frac{1}{K_m}$ where $n > m$

The empirical or computational procedure developed under this method shows the following brief table by calculating the sum of the reciprocals in numerical figures:

n	Dig	1/dig	CRAA-2
0	4	0,25000000000000000000	
1	21	0,04761904761904761904	0,29761904761904761904
2	143	0,00699300699300699300	0,30461205461205461204
3	1061	0,00094250706880301602	0,30555456168085762806
4	8363	0,00011957431543704412	0,30567413599629467218
5	68906	0,00001451252430847821	0,30568864852060315039
6	586081	0,00000170624879496178	0,30569035476939811217
7	5096876	0,00000019619861264037	0,30569055096801075254
8	45086079	0,00000002217979523125	0,30569057314780598379
9	404204977	0,00000000247399229821	0,30569057562179828200
10	3663002302	0,00000000027300010143	0,30569057589479838343
11	33489857205	0,00000000002985978691	0,30569057592465817034
12	308457624821	0,00000000000324193639	0,30569057592790010673
13	2858876213963	0,00000000000034978779	0,30569057592824989452
14	26639628671867	0,00000000000003753806	0,30569057592828743258
15	249393770611256	0,00000000000000400972	0,30569057592829144230
16	2344318816620308	0,00000000000000042656	0,30569057592829186886
17	22116397130086627	0,00000000000000004521	0.30569057592829191407

The result of the convergent series is a constant, a constant named CRAA-2 (see THE MATHEMATICAL CONSTANT OF AGUILAR-ACHA, Ciencia y Computación, EL DIARIO, La Paz – Bolivia, 22/VII/99).

Numerical Result

This new Aguilar-Acha's constant, CRAA-2, is originated in the former set of: a) a formal definition, b) expressions and formulas and c) a rigorous construction of the table up to 10^{18} decimal digits and also the primes comprised within each rank, associated to the advancement of the study of the theory of numbers, which represents to be the infinite value:

CRAA-2 = 0,30569057592829191…

CONCLUSION

The constant CRAA-2 is thus obtained and preliminary verified. In measurement terms this result suggests and help us in the decoding of primes' laws and/or rules related to its important regularities and is worthy of our attention concerning the outstanding and most important prime numbers, as objects or autonomous elements, in appearance just refractory to any kind of relations or regularities.

APPLICATIONS

Remembering that a beautiful and fundamental theorem states that: "all numbers can be written as a product of primes", already demonstrated by other mathematicians, the constant CRAA-2 help us to prove the conjecture of scarcity of the primes in the development of its succession or series and the density or distribution of primes. This constant is also useful to propose important theoretical new theorems relating prime numbers and the theory of the mathematical infinite. Which, undoubtedly, is of great importance for the study of the shape and construction of the structure of the mathematical science.

Also, in applied mathematics, to build algorithms for the design calculation of very long figures and the processing of chains, with precision and high security in computing machines with high capacity, speed and accuracy, to solve many modern complex problems, perform pure and applied investigations, notable classifications and hierarchizations, etc. useful to judge numerical propositions on the theory of numbers and other branches of pure and applied mathematics and related sciences.

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Annex

Proof of convergence:

let

$$\pi(x) = \text{\# positive primes less than x}$$

then

$$C_n = \pi\left(10^{n+1}\right) - \pi\left(10^n\right)$$

the theorem of the prime number

$$\lim_{x\to\infty} \frac{\pi(x)}{\frac{x}{\ln x}} = 1$$

on the other hand, let

$$d_n = \frac{10^{n+1}}{\ln 10^{n+1}} - \frac{10^n}{\ln 10^n}$$

we state that {C_n } and {d_n} are successions of the same order, then {1/C_n} and {1/d_n} are of the same order. Let`s prove it:

$$\lim_{n\to\infty} \frac{C_n}{d_n} = \lim_{n\to\infty} \frac{\pi\left(10^{n+1}\right) - \pi\left(10^n\right)}{\frac{10^{n+1}}{(n+1)\ln 10} - \frac{10^n}{n\ln 10}} = \lim_{n\to\infty} \frac{\frac{\pi\left(10^{n+1}\right)}{\left(\frac{10^{n+1}}{\ln 10^{n+1}}\right)} - \frac{\pi\left(10^n\right)}{\left(\frac{10^{n+1}}{\ln 10^{n+1}}\right)}}{1 - \left(\frac{n+1}{n}\right)\left(\frac{1}{10}\right)} =$$

$$= \lim_{n\to\infty} \frac{\frac{\pi\left(10^{n+1}\right)}{\left(\frac{10^{n+1}}{\ln 10^{n+1}}\right)} - \frac{\pi\left(10^n\right)}{\left(10\left(\frac{n}{n+1}\right)\left(\frac{10^n}{\ln 10^n}\right)\right)}}{1 - \left(\frac{n+1}{n}\right)\left(\frac{1}{10}\right)}$$

reordering, operating and canceling:

$$= \frac{1-\left(\frac{1}{10}\right)\left(\frac{n+1}{n}\right)}{1-\left(\frac{1}{10}\right)\left(\frac{n+1}{n}\right)} = 1$$

we observe that:

$$\frac{1}{d_n} = \frac{1}{\frac{10^{n+1}}{\ln 10^{n+1}} - \frac{10^n}{\ln 10^n}} = \frac{n(n+1)\ln 10}{n10^{n+1} - (n+1)10^n} = \frac{n(n+1)\ln 10}{(9n-1)10^n}$$

then

$\left\{\frac{1}{d_n}\right\}$ is of the same order than $\left\{\frac{n}{10^n}\right\}$, since

$$\lim_{n\to\infty} \frac{\left(\frac{1}{d_n}\right)}{\left(\frac{n}{10^n}\right)} = \lim_{n\to\infty} \frac{(n+1)\ln 10}{9n-1} = \lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)\ln 10}{9-\frac{1}{n}} = \ln\frac{10}{9} \in \Re^+$$

since

$\sum_{n=1}^{\infty} \frac{n}{10^n}$ is convergent, it follows that $\sum_{n=1}^{\infty} \frac{1}{d_n}$ is convergent,

consequently

$$\sum_{n=1}^{\infty} \frac{1}{C_n} \text{ is convergent} \qquad \textbf{Q.E.D.}$$